

Problem Set I
Tourism in Developing Countries (MTEE)

Due date: Wednesday, March 17th.

1. Consider a family composed by identical members ($c_1 = c_2 = c$ and $l_1 = l_2 = l$). The utility function is given by $U(c, l) = c^a l^{1-a}$, $a \in [0, 1]$ where c is consumption and l is leisure. The household runs a family business and produces an output using the following technology, $F(L, A) = L^b A^c$, where L is labour and A is land. Markets are complete.
 - a) Describe the household's problem. Represent the 'full-income' constraint.
 - b) Does the separation hypothesis hold? Why?
 - c) Obtain the values of L^* , A^* and $F(L^*, A^*)$.
 - d) Suppose that the 'full-income' is M . Describe how this household maximize the utility function (obtain c^* and l^*).
2. Consider the previous model but now assume that households cannot hire labour for a wage.
 - a) What would be the 'full-income constraint'? Is the 'full-income' larger or lower than in the previous problem?
 - b) Obtain the values of L^* , A^* . And given M , obtain c^* and l^* .
3. Consider the model presented in exercise 2. There is no labour market, but households are able to hire and rent land in the market. However, there is a government regulation that limits the amount of land supplied to the market. Let us call Z the maximum amount of land the household can offer to the land market. Discuss how this regulation would affect household's optimal choices.

4. Consider an economy with two sectors (a Lewis' type economy). The production function for the two goods is given by $Q_i = A_i L_i^\alpha$ being $i = (a, m)$ and $\alpha \in (0, 1)$. A is a productivity parameter equal in both sectors of the economy.
- Obtain the expressions of V_a , M_a and $E(Q_a, L_a)$. Which one is higher V_a or M_a ?
 - Assume that industrial wages must compensate for the forgone average income earning at the family farm, so that $w = pV_a$. Let us assume that individuals have Cobb-Douglas preferences given by: $U(C_a, C_m) = C_a^b C_m^{1-b}$, $b \in [0, 1]$. Obtain the demand function of $C_a(y, p)$.
 - Assume that the economy is closed. Obtain the expressions of $E(w, L_a)$ and $E(p, L_a)$ in equilibrium. How is contributing the lowering of the agriculture labour share to the economy? Why?
 - Is the income elasticity demand for agricultural product different than the one for manufacturing goods?
5. Consider the economy depicted in the previous exercise in equilibrium. Now, the agricultural productivity improves, *ceteris paribus*. What is the resulting effect on the agricultural output? And what would be the effect on the agricultural labour share?
6. Assume that individuals have Stone-Geary preferences, $U(C_a, C_m) = b \ln(C_a - \gamma) + \ln C_m$, with $\gamma > 0$ (minimum consumption). The economy has two sectors similar to the one described in the problem (5). Industrial wages are set according to $w = pV_a$.
- Obtain the demand function of $C_a(y, p)$. What is the consumption level when income is null?
 - Assume that the economy is closed. Obtain the expressions of $E(w, L_a)$ and $E(p, L_a)$ in equilibrium. How is contributing the lowering of the agriculture labour share to the economy? Why?
 - Is the income elasticity demand for agricultural product different than the one for manufacturing goods?
 - What is the resulting effect of a productivity improvement in the agricultural sector on the rural output? And what would be the effect on the agricultural labour share?