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THE EXCITATION AND DAMPING OF TRANSVERSAL CORONAL LOOP OSCILLATIONS

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ABSTRACT

The excitation and damping of transversal coronal loop oscillations is studied using a simple line-tied one-dimensional model. The dynamics of the loop and the coronal environment is governed by the well-known Klein-Gordon equation, and so a localized disturbance (representing the effect of a solar flare) gives rise to a perturbation that undergoes dispersion as it propagates toward the loop. As a consequence, the loop oscillates at the external cutoff frequency, and its motion attenuates with time roughly as $t^{-1/2}$ (with dense and wide loops having even stronger damping profiles). Hence, the damping of transversal loop oscillations is not related to any dissipation mechanism but is simply produced by the wake of the traveling disturbance. In addition, these damped oscillations are not related to the kink mode, although this mode can be excited after the attenuation process by the energy of the wave packet deposited in the loop.

Subject headings: MHD — Sun: corona — Sun: magnetic fields — waves

1. INTRODUCTION

Recently, *TRACE* observations have allowed us to study in detail transversal coronal loop oscillations excited impulsively (Aschwanden et al. 1999, 2002; Nakariakov et al. 1999; Nakariakov & Ofman 2001; Schrijver et al. 2002). In most of the observations oscillations are triggered by a nearby flare (see Hudson & Warmuth 2004) and produce transversal loop motions. Such oscillations have been interpreted in terms of the standing fast kink magnetohydrodynamic (MHD) mode, since they produce transversal displacements of the loop axis. The understanding of these oscillations can be used as a diagnostic tool for the magnetic field (Nakariakov & Ofman 2001; Nakariakov 2004) and to provide seismic information about the coronal conditions (Roberts et al. 1984; Roberts 2004).

An interesting feature of oscillating loops is that the amplitude of oscillation has been found to be strongly attenuated (Nakariakov et al. 1999; Aschwanden et al. 2002). Several theoretical damping mechanisms have been proposed, phase mixing and resonant absorption being the most popular (see Aschwanden et al. 2003 and references therein).

In general, most of the theoretical studies on loop oscillations assume that the system is in its stationary state and that the loop dynamics is given by the normal modes. From such kind of analysis, a lot of features have been described in detail (Spruit 1982; Edwin & Roberts 1982, 1983; Cally 1986, 2003). However, normal modes do not represent the whole picture. Only a few works have dealt with the time-dependent problem of the excitation of loop oscillations. Murawski & Roberts (1993a) analyzed the temporal evolution of driven waves in loops modeled by smoothed slabs of enhanced density, and they discussed some features of kink and sausage oscillations. Murawski & Roberts (1993b, 1993c) studied the temporal signatures of impulsively generated waves in two-dimensional slabs.

Recently, Uralov (2003) has studied the effect of a propagating wave generated by a localized disturbance on loops. This author has pointed out that the wake of the traveling wave packet could be responsible for the rapid damping of the observed oscillations. Nevertheless, the loop structure was not included in his study.

The purpose of this Letter is to provide further insight into the excitation and time evolution of waves in line-tied coronal loops. We are interested in the details of propagating features produced by localized events and their influence on loop oscillations.

2. RESPONSE OF A LINE-TIED UNIFORM MEDIUM TO A FLARE EVENT

The plasma is permeated by a straight and uniform magnetic field ($\boldsymbol{B} = B_0 \boldsymbol{e}_z$) extending between $z = -L/2$ and $z = L/2$, with L being the length of the magnetic field lines. In the β zero limit the ideal linearized MHD equations reduce to the well-known Klein-Gordon equation for the fast modes,

$$\frac{\partial^2 \xi}{\partial t^2} = v_{\rm A}^2 \frac{\partial^2 \xi}{\partial x^2} - \omega_c^2 \xi, \tag{1}$$

where x is the coordinate in direction normal to the magnetic field lines, ξ is the normal displacement, $v_{\rm A} = B_0/\sqrt{\mu\rho}$ is the Alfvén speed, and $\omega_c = v_{\rm A} k_z$ is the cutoff frequency. We have assumed that perturbations are of the form $e^{-ik_z z}$, which allows us to study the effect of line-tying by selecting the appropriate value of k_z (for the fundamental mode $k_z = 2\pi/2L$). Hereafter, we use dimensionless variables: length is normalized to L, velocity to $v_{\rm A}$, and time to the Alfvén transit time $\tau_{\rm A} = L/v_{\rm A}$. Normal mode solutions of this equation (i.e., with a dependence of the form $e^{i(\omega t - kx)}$) yield the dispersion relation $\omega = (k^2 v_{\rm A}^2 + \omega_c^2)^{1/2}$, which indicates that wave propagation is dispersive. The propagation speed for waves with wavenumber k is the group velocity,

$$v_g = \frac{d\omega}{dk} = \frac{k v_{\rm A}^2}{\sqrt{k^2 v_{\rm A}^2 + \omega_c^2}} = \frac{\sqrt{\omega^2 - \omega_c^2}}{\omega} v_{\rm A}, \tag{2}$$

which ranges between zero, for waves with infinite wavelength, and $v_{\rm A}$, for waves with infinitely small wavelength. Therefore, modes with short wavelengths propagate faster than modes with larger wavelengths. In addition, any mode with frequency below the cutoff frequency is evanescent and unable to propagate in the medium. The dispersive behavior of waves is simply due to the fact that we assume a fixed wavelength along the loop to model the effect of fixed footpoints.

A simple analytical solution of the time-dependent problem (eq. [1]) can be derived for the following initial conditions:

$$\xi(x,0) = 0, \qquad \frac{\partial \xi}{\partial t}(x,0) = v_0 \delta(x), \tag{3}$$

with v_0 being the velocity of the initial perturbation. The solution in terms of the Bessel function J_0 is given by

$$\xi(x,t) = \frac{v_0}{4\omega_c} J_0\left[\omega_c \sqrt{t^2 - (x/v_{\rm A})^2}\right] \mathcal{H}(t - x/v_{\rm A}), \tag{4}$$

where $\mathcal{H}$ is the Heaviside function. This solution, studied by Lamb (1908; see also Rae & Roberts 1982), is characterized by a wave front traveling at the Alfvén speed in the direction normal to the magnetic field lines and by a wake behind this pulse oscillating at the cutoff frequency ω_c (see dotted line in Fig. 1). For $t \gg x/v_{\rm A}$ the asymptotic expression for the displacement is

$$\xi(x,t) = \frac{v_0}{\sqrt{8\pi}\,\omega_c^{3/2} t^{1/2}} \cos(\omega_c t - \pi/4), \tag{5}$$

which indicates that the displacement amplitude decays as $t^{-1/2}$ and that the frequency of oscillation is simply ω_c. These results are different from those obtained by Uralov (2003) because we describe free oscillations in an unbounded medium by solving the inhomogeneous initial value problem while Uralov solved an inhomogeneous boundary value problem in a bounded system by imposing that the velocity must be zero at infinity. The solution presented here appropriately describes the physical problem of a flare-induced perturbation and the propagation of the resulting disturbance in the x-direction.

It is interesting to consider the effect of a wave source of finite width, instead of a δ-function perturbation. We consider for simplicity a perturbation with a Gaussian profile

$$\xi(x,0) = 0, \qquad \frac{\partial \xi}{\partial t}(x,0) = v_0 \exp(-x^2/a^2), \tag{6}$$

with a being the width of the disturbance. A formal solution of equation (1), together with the initial conditions given by equation (6), is derived following Whitham (1974; see also Rae & Roberts 1982),

$$\xi(x,t) = \frac{v_0 a}{\sqrt{\pi}} \int_0^\infty \frac{\exp(-k^2 a^2/4)}{\omega} \sin \omega t \cos kx \, dk, \tag{7}$$

where ω depends on k through the dispersion relation. The integral in equation (7) must be evaluated numerically.

For a flare event such as the one described in Aschwanden et al. (1999) the spatial scale of the compact flare source is much smaller than the mean length of the oscillating loops; hence, values of a below $0.1L$ are close to realistic source widths. In Figure 1 we have represented the solution given by equation (7) for $a = 0.1L$. We can see that for large t, the solution tends to the results of the δ-function perturbation and that the differences are mainly in the wave front but not in the oscillating wake. Hence, an excitation with a Gaussian shape does not change much (for a reasonable range of disturbance widths a) the displacement profile in comparison with the δ-function perturbation.

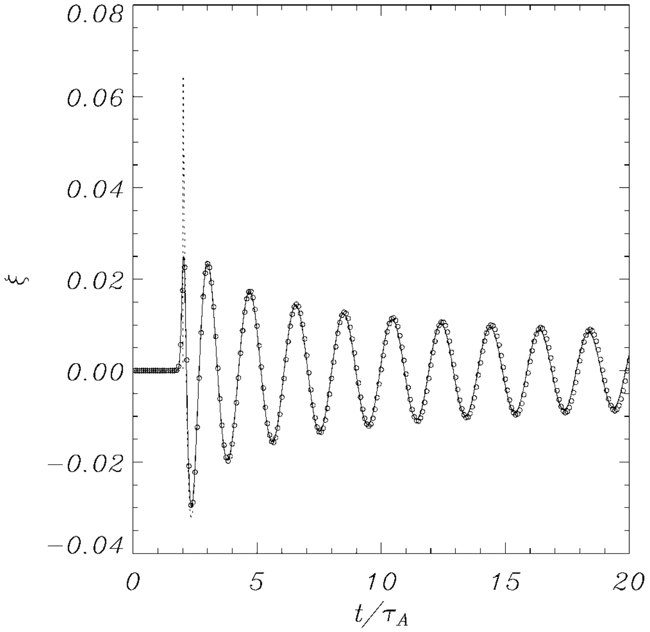

FIG. 1.—Displacement as a function of time at position $x = 2L$ for different wave source widths; the dotted line corresponds to a δ-function, and the solid line corresponds to a Gaussian profile (see eq. [6]) with $a = 0.1L$. The numerical solution with a Gaussian profile with $a = 0.1L$ for a thin, low-density slab model ($d = 0.01L$ and $\rho_{\rm in}/\rho_{\rm out} = 2$) is displayed with circles. For comparison purposes all the profiles have been normalized to the same amplitude. It is obvious that different kinds of impulsive excitation and the presence or nonpresence of the loop result in essentially the same temporal behavior for this low-density contrast, thin loop.

3. EFFECT OF THE LOOP STRUCTURE

The previous model does not take into account the effect of the dense loop, but it simply describes the features of an impulsively generated perturbation in a uniform, dispersive medium. However, active regions are characterized by the presence of loops with enhanced density, and it is precisely these structures that clearly show transversal oscillations. We consider here the effect of structuring on wave propagation with a more realistic model for which the following density profile is assumed:

$$\rho = \begin{cases} \rho_{\rm in} & \text{if } |x| \le d, \\ \rho_{\rm out} & \text{if } |x| > d, \end{cases} \tag{8}$$

where d is half the slab width. Since we assume a uniform magnetic field, the Alfvén velocity changes from outside ($v_{\rm A}$) to inside the slab ($v_{\rm As}$). Normal modes of this configuration have been studied in detail by Edwin & Roberts (1982).

It is possible to explain qualitatively the effect of a traveling disturbance impacting laterally on a loop from the analysis of the transmission coefficient of the slab-corona configuration (see eq. [20] in Uralov 2003). The response of the loop to the pulse depends on the spectral components of the wave packet and on the loop parameters. According to the transmission coefficient the slab is basically transparent to short wavelengths, traveling fast and forming the front of the disturbance. On the other hand long wavelengths, and specially those components with frequency tending to ω_c, are strongly reflected from the loop. Note that frequencies close to ω_c are precisely those responsible for the shape of the wake. Besides transmission and reflection of waves, wave trapping is also possible; i.e., some energy can be deposited in the loop. To study this phenomenon it is necessary to solve the time-dependent problem, which is now more complex since the Alfvén speed now changes from inside to outside the slab. For this reason, we have solved equation (1) numerically. We have used the finite element code PDE2D (Sewell 2000) and have checked that the artificial diffusion introduced by the numerical scheme is small. This is also a critical point since we are interested in the analysis of damping.

Since the slab is located around $x = 0$ we have generated a perturbation in the external medium (corona) with the same shape as in equation (6), but at a distance x_0 from the center of the slab. The perturbation initially produces the displacement profile studied in the previous section, but at time $\sim x_0/v_{\rm A}$ (the wave front travels at the Alfvén speed) the pulse reaches the slab and starts to interact with the dense layer. In order to have realistic values of the density contrast, loop width, and loop length, we have considered the numbers given by Aschwanden et al. (2002, 2003). From the analysis of the oscillating loops, the previous parameters have typical values in the range $2 \leq \rho_{\rm in}/\rho_{\rm out} \leq 5$ and $0.02L \leq d \leq 0.05L$, with mean values $\rho_{\rm in}/\rho_{\rm out} = 3$ and $d = 0.04L$.

In Figure 1 we have plotted with circles the displacement as a function of time at the center of the slab ($x = 0$) for a situation with a wave source at a distance $x_0 = 2L$ from the slab, a slab half width $d = 0.01L$, and a density contrast $\rho_{\rm in}/\rho_{\rm out} = 2$. It is clear that the difference between the homogeneous model and the slab model is simply a very slight phase shift with time. Apart from this small shift, the overall displacement is quite similar. This indicates that for the considered parameters (i.e., for a low-density contrast, thin structure), the loop oscillates and decays according to the wave passing through it, which is only slightly affected by the presence of the loop. In Figure 2*a* we have plotted the displacement as a function of time at the center of the slab for a thicker ($d = 0.03L$) and denser ($\rho_{\rm in}/\rho_{\rm out} = 5$) loop. The results are now quite different from those presented in Figure 1. First, the displacement has a faster attenuation with time than the $t^{-1/2}$ law found in the uniform case. This faster damping time with respect to the homogeneous case can be understood with the dependence of the transmission coefficient on ω. The initial disturbance produces a signal with periods that increase with time at a fixed position; i.e., ω decreases with time. As ω decreases, reflection increases and results in a lower efficiency of energy transmission to the loop with time that leads to the faster attenuation profile. Second, the frequency of oscillation decreases considerably. In order to study this issue in more detail we have calculated the wavelet transform of the signal (see Torrence & Compo 1998). In Figure 2*b* we can see that, because of the damping, the power decreases with time while the frequency smoothly changes from values above the external cutoff (*dashed line*) to frequencies below it. In this figure we have represented with a solid line the eigenfrequency of the fundamental kink body mode of the slab. We clearly see that for large times, the power concentrates around the kink mode eigenfrequency, which is close to the external cutoff since we are in a regime of small loop width in comparison with the loop length (see Edwin & Roberts 1982).

Hence, the physical interpretation of the time signature of the displacement at the center of the loop is the following: initially the slab behaves as driven by the wave, next the effect of the wake on the loop decreases as a result of the increased reflection from the slab, and finally the loop starts to oscillate with its natural (kink mode) frequency. The amplitude of oscillation of the eigenmode depends on the amount of energy deposited in the loop during the initial phase, which in turn depends on the loop width and density, on the width of the initial perturbation, and on its distance from the slab. A quantitative study of the amount of energy deposited in the loop and its dependence on the previous parameters is outside the scope of this Letter.

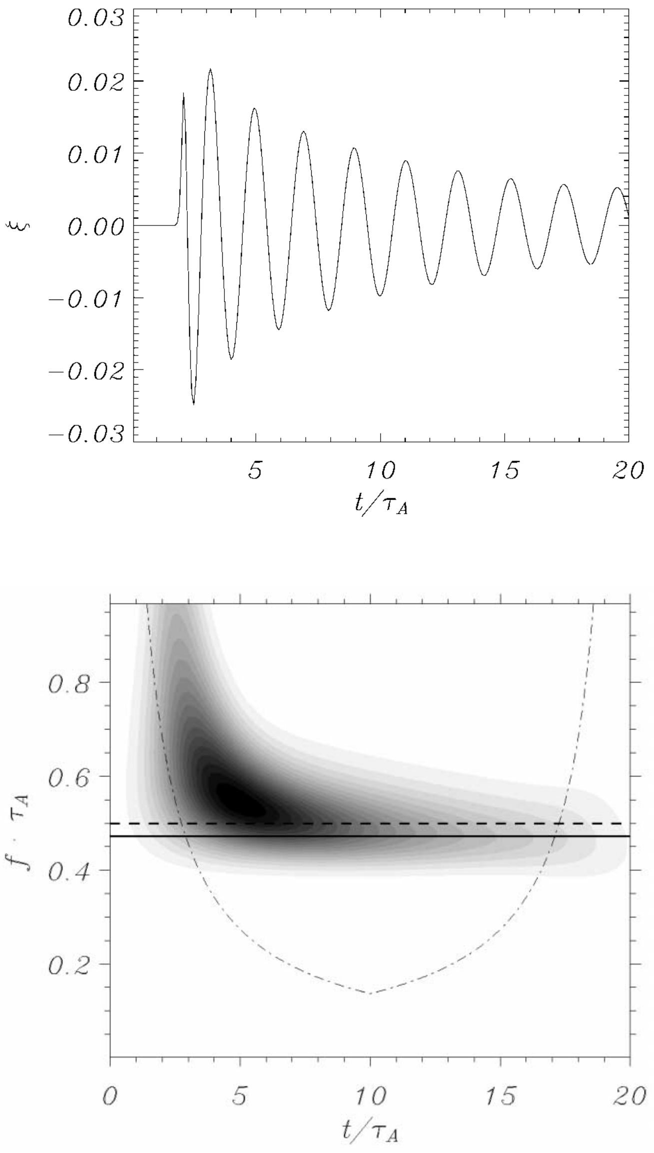

FIG. 2.—(*a*) Displacement as a function of time at position $x = 0$ (center of the slab). The loop parameters are $d = 0.03L$ and $\rho_{\rm in}/\rho_{\rm out} = 5$, the initial perturbation is generated at $x_0 = 2L$, and the width of the initial pulse is $a = 0.1L$. (*b*) Wavelet transform (calculated with the Morlet mother function) of the signal shown in (*a*). The dashed and solid lines represent the external cutoff frequency ($f_c = \omega_c/2\pi = v_{\rm A}/2L$) and the kink mode eigenfrequency, respectively. The dash-dotted line represents the cone of influence in the wavelet transform.

4. CONCLUSIONS

We have shown several features of transversal loop oscillations with simple one-dimensional simulations. The damping of loop oscillations, which should be understood as a global process and not restricted to individual loops or magnetic field lines (see also Uralov 2003 and Hudson & Warmuth 2004), is simply produced in our model by the dispersive nature of the traveling pulse and is not related to any dissipation mechanism.

The dynamics of the transversal oscillations of thin and not too dense loops is dominated by the wave passing through them. The amplitude of the loop displacement attenuates basically as $t^{-1/2}$ and oscillates with the cutoff frequency. Denser and wider loops show stronger attenuation profiles depending on the distance to the wave source. After a phase driven by the wave wake, the loop oscillates with the eigenfrequency corresponding to the fundamental kink mode. Note that the existence of the transitory phase has been overlooked in other studies, in which it has been interpreted that the loop oscillates instantaneously with the kink mode after the wave reaches the loop position. The case presented here (Fig. 2) shows that the effect of the traveling disturbance interacting with the loop is a mechanism that naturally leads to attenuation of the amplitude of oscillation. However, we do not pretend to compare the damping times obtained here with those of the observed loop oscillations since our model is too simple.

One should expect some differences if a cylindrical loop is studied instead of a slab model. For such geometry normal modes are much more confined than for the Cartesian slab (Díaz 2004), which can have implications regarding wave trapping. Still more important, the period of the kink mode $2L/c_k$ (see Edwin & Roberts 1983) is not close to the external cutoff frequency, which indicates that the differences between the period of the transient and the period of the normal mode should be accentuated and, in consequence, more easily detected. Up to now, *TRACE* observations of loop oscillations do not have enough temporal cadence to detect small variations in the period (in general, there are only four or five samples per period). When higher temporal resolution data become available, a wavelet transform with similar characteristics to the one presented here should provide us with a clear indication of the efficiency of the mechanism studied in this work.

Finally, note that since we have considered one-dimensional simulations (we have assumed a fixed k_z) the different phases of impulsively generated waves along slabs, studied by Roberts et al. (1984) and Murawski & Roberts (1993a, 1993c), are not present in our study. However, is interesting to investigate the combined effect of propagation across and along a loop with fixed ends, i.e., to study the two-dimensional problem.

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REFERENCES

Aschwanden, M. J., De Pontieu, B., Schrijver, C. J., & Title, A. M. 2002, Sol. Phys., 206, 99

Aschwanden, M. J., Fletcher, L., Schrijver, C. J., & Alexander, D. 1999, ApJ, 520, 880

Aschwanden, M. J., Nightingale, R. W., Andries, J., Goossens, M., & Van Doorsselaere, T. 2003, ApJ, 598, 1375

Cally, P. S. 1986, Sol. Phys., 103, 277

———. 2003, Sol. Phys., 217, 95

Díaz, A. J. 2004, Ph.D. thesis, Univ. Illes Balears

Edwin, P. M., & Roberts, B. 1982, Sol. Phys., 76, 239

———. 1983, Sol. Phys., 88, 179

Hudson, H. S., & Warmuth, A. 2004, ApJ, 614, L85

Lamb, H. 1908, Proc. London Math. Soc., Ser. 2, 7, 122

Murawski, K., & Roberts, B. 1993a, Sol. Phys., 143, 89

———. 1993b, Sol. Phys., 144, 101

———. 1993c, Sol. Phys., 144, 255

Nakariakov, V. M. 2004, in *SOHO* 13—Waves, Oscillations and Small-Scale Transients Events in the Solar Atmosphere: A Joint View from *SOHO* and *TRACE*, ed. H. Lacoste (SP-547; Noordwijk: ESA), 407

Nakariakov, V. M., & Ofman, L. 2001, A&A, 372, L53

Nakariakov, V. M., Ofman, L., DeLuca, E. E., Roberts, B., & Davila, J. M. 1999, Science, 285, 862

Rae, I. C., & Roberts, B. 1982, ApJ, 256, 761

Roberts, B. 2004, in *SOHO* 13—Waves, Oscillations and Small-Scale Transients Events in the Solar Atmosphere: A Joint View from *SOHO* and *TRACE*, ed. H. Lacoste (SP-547; Noordwijk: ESA), 1

Roberts, B., Edwin, P. M., & Benz, A. O. 1984, ApJ, 279, 857

Schrijver, C. J., Aschwanden, M. J., & Title, A. M. 2002, Sol. Phys., 206, 69

Sewell, G. 2000, Finite Differences, Finite Elements and PDE2D (El Paso: Univ. Texas)

Spruit, H. C. 1982, Sol. Phys., 75, 3

Torrence, C., & Compo, G. P. 1998, Bull. Amer. Meteorol. Soc., 79, 61

Uralov, A. M. 2003, Astron. Lett., 29, 486

Whitham, G. B. 1974, Linear and Nonlinear Waves (New York: Wiley)