



## **Masters in Mathematics**

### **Subject Description**

#### **Technical File**

##### **Course/Subject**

Name: *An Introduction to Mathematical Processing of Fuzzy Information*

Code: 10516

Type: (obligatory or elective) Elective

Semester: (one or two): One

Timetable:

Language: Catalan

##### **Lecturers**

###### **Head Lecturer**

Name: Gaspar Mayor Forteza

Contact Details: 2983

###### **Other Lecturers**

Name: Joan Torrens Sastre

Contact Details: 3195

Name:

Contact Details:

**Number of Credits: 5**

#### **Objectives and Skills**

(General/specific subject or course objectives and skills)

##### **Aims:**

- O1: Introducing students to the world of Fuzzy Logic and Information Aggregation.
- O2: Providing basic concepts to the material both from a theoretical viewpoint and applications.
- O3: Providing the necessary base for students to be able to start scientific research in this field.

##### **General Skills:**

- G1. Understanding, analysing, detailing and posing problems in mathematical language.
- G2. Mastering oral and written mathematical language.
- G3. Acquiring a good ability to reason and argue.
- G4. Learning to communicate and work in a group.
- G5. Acquiring the ability to think creatively and with initiative.
- G6. Acquiring the ability to learn independently.
- G7. Knowing how to search, research and study a research topic in-depth.
- G8. Acquiring the ability to research independently.
- G9. Knowing how to handle specialised bibliography.
- G10. Knowing how to find, record and synthesise information from different sources.

##### **Specific Skills:**

- E1. Understanding and handling basic concepts and results for t-norms.

- E2. Understanding and knowing how to use continuous t-norm classification.
- E3. Knowing how to solve certain types of functional equations.
- E4. Understanding t-norm application fields.
- E5. Understanding and handling different types of operators involved and their use in approximate reasoning and mathematical morphology.
- E6. Understanding different t-norm generalisations. t-norm step in aggregate function.
- E7: Uninorms and t-operators.
- E8: Understanding fuzzy logic as multi-valued logic. Analysing connectors.
- E9: Understanding different types of fuzzy logic.
- E10. Understanding and handling the basic concepts and results to the theory of fuzzy sets and fuzzy relations.
- E11. Understanding and handling the compositional rule of inference.
- E12. Understanding and using additive and non-additive fuzzy measures.
- E13. Understanding the basic definition and properties to fuzzy measures.
- E14. Understanding the most important applications of fuzzy sets and fuzzy logic.
- E15. Understanding the use in approximate reasoning and the theory of fuzzy control.

## Contents

(itemised course/subject contents)

## SYLLABUS

### Part I Triangular Norms (t-norms)

#### 1 Basic Definitions, Examples and Properties

- Definitions and examples. Order associated to a t-norm. Continuity.
- Notable elements: idempotents, zero divisors, nilpotents.
- Archimedean t-norms: strict and nilpotents.
- Negations and duality. De Morgan's Theorem. The principle of non-contradiction and excluded middle.

#### 2. Continuous Triangular Norms

- Isomorphisms for t-norms.
- Additive and multiplying generators.
- Characterisation of strict and nilpotent t-norms.
- Ordinal sums. Classification of continuous t-norms.
- Discretisation for t-norms.

#### 3 Triangular Norms and Functional Equations

- Elemental techniques for resolving functional equations. Cauchy's equation.
- Characterisation of the four basic t-norms via functional equations.
- An analysis of complementary properties to t-norms and t-conorms via functional equations.
- Frank's equation. Related equations.
- Functional equations and t-norm families: Frank's t-norms, Aczél-Alsina t-norms, Mayor-Torrens' t-norms, Hamacher's t-norms.

#### 4 Related Operations

- Generalisations for t-norm concept: Aggregation functions.
- Basic concepts and properties.
- Aggregation functions defined from t-norms.
- Uninorms and t-operators (nulnorms).
- Copula

## Part II Fuzzy Sets and Fuzzy Logic

### 5 Fuzzy Sets

- Uncertainty and fuzziness. Fuzzy mathematical models.
- Fuzzy Sets. Operations with fuzzy sets.
- Fuzzy quantities. Fuzzy numbers. Fuzzy intervals.

### 6 Classical Logic to Fuzzy Logic

- Classic trivalued logic.
- Trivalued logic.
- Fuzzy logic.
- Fuzzy logic and Lukasiewicz logic.

### 7 Operators Involved and Approximate Reasoning

- Operators involved in approximate reasoning.
- Implicators defined from t-norms and t-conorms: R-implications, S-implications and QL-implications.
- Modus-ponens. Applications to mathematical morphology and image processing.

### 8 Other Applications

- Expert systems.
- Fuzzy control.
- Fuzzy data analysis.
- Taking decisions in fuzzy environments.
- Bioinformatics.

## Teaching and Learning Methodology

(itemised: attended/individual work, virtual learning use, group types, etc.)

Learning Methodology	Group Types	Work Types	Student Hours
Classes Attended	Large group	Attended	20
Group work presentation	Large group	Attended	2
Practical Classes	Medium group	Attended	8
Tutorial	Small group	Attended	2
Evaluation	Individual	Attended	4
			36 attended
Theoretical and Practical Work	-	Individual	24
Theoretical study	-	Individual	30
Practical Study	-	Individual	35
			89 individual
			Total = 125

## Evaluation Criteria and Procedures

(itemised: assessment criteria and procedures, assessment tools and qualification criteria)

EVALUATION CRITERIA AND PROCEDURES
<b>Evaluation Criteria</b> Achievement level for subject skills (general and specific). Acquisition of theoretical knowledge, the ability to apply it in problem solving and clear oral and written exposition will be taken into account.
<b>Evaluation Tools</b> -Oral and written presentation of a theory and/or practical work (T) -Examination (E).
<b>Marking Criteria</b> The examination will be worth 60 per cent and the theory/practical work the other 40 per cent. i.e. Final mark = $0.6E + 0.4T$ .

## Bibliography, Recommended Reading and Other Didactic Resources for Individual Work

(itemised, at least, the information relating to the relevant and updated bibliography)

### Basic Bibliography:

- E.P. Klement, R. Mesiar, E. Pap, Triangular norms, Kluwer Academic Publishers, Dordrecht, 2000.
- T. Calvo, G. Mayor, R. Mesiar (editors), Aggregation operators. New trends and applications, Studies in Fuzziness and Soft Computing, 97, Physica Verlag, Heidelberg, 2002.
- J. Aczél, Lectures on functional equations and their applications, Academic Press, New York, 1966.
- B. Schweizer, A. Sklar, Probabilistic metric spaces, North Holland, New York, 1983.
- R.B. Nelsen, An introduction to copulas, Springer-Verlag, New York, 1999.
- G.J. Klir and Bo Yuan, Fuzzy Sets and Fuzzy Logic (Theory and Applications), Prentice Hall, New Jersey, 1995.
- J. Fodor and M. Roubens, Fuzzy Preference Modelling and Multicriteria Decision Support, Kluwer Academic Publishers, Netherlands, 1994.
- H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer Academic Publishers, Netherlands, 1996.
- H. T. Nguyen, E. A. Walker, A First Course in Fuzzy Logic, Chapman & Hall, 2006.
- C. Alsina, M.J. Frank and B. Schweizer, Associative functions. Triangular norms and copulas, World Scientific, Singapore, 2006.